Math 128a – Week 4 Worksheet
GSI: Izak, (2/10/21)

2.3 Problems

Problem 1. *Come up with a function $f \in C^2[a,b]$ with $f(p) = 0$ for some $p \in [a,b]$ such that Newtons method fails to converge for any initial guess not equal to p.*

Problem 2. *Derive the error formula for Newton's method:*

$$|p - p_{n+1}| \leq \frac{M}{2|f'(p_n)|}|p - p_n|^2$$

2.4 Problems

Problem 3. *Generalize one of your homework problems. Construct a sequence p_n converging to p at order α with asymptotic error constant λ.*

2.5 Problems

Problem 4. *Steffensen's method is applied to a function $g(x)$ using $p_0^{(0)} = 1, p_2^{(0)} = 3$ to obtain $p_0^{(1)} = .75$. What is $p_1^{(0)}$?*

Problem 5. *Prove that if p_n converges linearly to p and $\lim_{n\to\infty} \frac{p_{n+1}-p}{p_n-p} < 1$, then $\lim_{n\to\infty} \frac{\hat{p}_n-p}{p_n-p} = 0$ where $\hat{p}_n$ comes from Aitken's Δ^2 method. (Hint: let $\delta_n = (p_{n+1} - p)/(p_n - p) - \lambda$ and show that $\lim_{n\to\infty} \delta_n = 0$. Then express $(\hat{p}_{n+1} - p)/(p_n - p)$ in terms of δ_n, δ_{n+1} and λ).*